A Beginner's Guide to Using Computer Antenna Modeling Programs

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When MININEC became available to antenna experimenters as an antenna modeling computer program, they absorbed it with relish. It saved them hours, if not days, of futile construction effort on designs that would not improve performance. Now the program is available in at least three versions for the IBM PC and compatible computers (MININEC3, MN, and ELNEC[1]) to the average ham at reasonable costs. Depending upon the version, it will run on PCs with or without math coprocessors. Two versions (MN and ELNEC) produce excellent screen graphics of antenna patterns, along with documentation of the design, source impedance factors, and current distribution.

Are these programs really useful to the beginning and moderately experienced ham? The answer is a resounding YES! When used within their limitations, these programs can go far beyond textbooks in teaching us why our antennas act the way they do.[2] They can also help us make better decisions on what antennas to build or buy and how to mount them. However, after a brief period of sampling the test designs included with the program, the antenna modeling program may end up in a disk file box. The reason for discarding these valuable programs is that most of us fail to understand all they can tell us, and that—in turn—is because we do not set up procedures to squeeze meaningful information out of the program.

The purpose of this article is to show the beginner how to start using an antenna modeling program effectively. A good beginning requires three areas of effort: (1) setting up certain program basics, (2) setting up consistent modeling conventions, and (3) developing a baseline of information about basic antennas located on one's own property. The first step permits us to focus on and master the essentials of the program, saving advanced features for later. Step 2 allows us to model accurately and confidently, with minimal error in comparing one design with another. Step 3 allows us to interpret intelligently the patterns that emerge from new designs we try. Once we have mastered both the program and what it can tell us, we can expand our knowledge by using its advanced features.

The suggestions presented here are no substitute for mastering the instructions that come with the program; instead, they are designed to supplement those instructions. The aim here is to make the program and its procedures as useful and instructive as possible, even for the beginning antenna modeler. The program I use is ELNEC, but the steps suggested here can be translated for use with nearly any MININEC-based program.

Setting Up the Program

Advanced users of ELNEC and similar programs require considerable flexibility, so the programs offer many options to the user. Often these options inhibit the beginning student of antennas by offering choices among which the user cannot decide. Therefore, the first step in getting the most out of the antenna modeling program is to make decisions, even if they are initially made for weak reasons. For the new user, convenience may be the best reason available.

ELNEC offers a menu with many options, only a few of which we need for the purpose of getting used to the program. The "Wires" entry is for describing the antenna we shall model. The "Sources" entry is also crucial to each model, telling the program where the antenna is fed. Most of the basic antennas we shall start with—dipoles, verticals, Yagis, and the like—use only one source. Until we start modeling trap dipoles and beams, we shall likely have no use for the "Loads" entry (resistances and reactances as part of the antenna), so we may leave it at 0. Other fields to leave at default initially are the "Analysis Resolution" entry (1°), the "Step Size" entry (5°), and the "Field(s) to Plot" entry (total field only). After we become more advanced or develop special interests in the program, we may wish to alter those entries.

Two entries where we must make an initial decision are "Units" and "Ground Type." For US hams, feet and inches are the most commonly encountered antenna measurements. For HF work, start and stick with measurements in feet. This will ease the problem of calculating specific dimensions of antennas by sending us through the same steps with each antenna.

The selection of the type of ground to use is a bit more complex to decide. The program offers free space, perfect ground, and real-ground choices. My personal preference is for real ground. The default real-ground description uses average soil (as explained in the manual). Unless there is good reason to change this entry, initially stick with it. However, if you know the electrical characteristics of your soil and surrounding terrain (and water bodies, if present), it may pay to go through the setup procedures in the manual for establishing a detailed ground description. Remember, however, that the program does not account for yard clutter that may affect an antenna. Later you can compare real-ground patterns with patterns over perfect ground to see what differing soil conditions can do to an antenna's gain and pattern. You may also wish to compare your real-ground patterns with the free-space patterns so common in textbooks. Those exercises will be enlightening, but for comparing one antenna with another, stay with a consistent description of the ground. There are program limitations that the selection of real ground cannot overcome; we shall note the most significant one for the beginning modeler later.

Only two more menu entries require comment at this point. For the antenna "Title," choose your words and abbreviations carefully to pack in as much information as possible. That will allow you to easily distinguish one antenna design from another. "Dipole" is not a very good choice for a title, but "10 M Wire Dipole, 30 Ft Up" might be. The "Frequency" entry also requires care. For each band on which you compare different antennas, use the same frequency or set of frequencies. Do not shift from one end of a band to the other when changing antenna designs (unless you have a special reason for doing so); your comparisons may not be valid. If necessary, evaluate an antenna design at two or three frequencies within a band, using the same frequencies for all antenna designs.

Having now decided most of the menu entries in advance, we have reduced the remaining entries to a manageable few. To display and print out our results, we shall deal with decisions involving "Plot Type" and "Azimuth/Elevation Angle" later on. First we have to model our antenna design in order to enter it into the computer, and that takes some special forethought.

Setting Up Modeling Conventions

Getting good results from an antenna program begins with pencil and paper. Whether you are modeling your own antenna, an idea from a handbook, or an experimental design, you will have to put the figures on paper and change their form to what the program wants to see. Therefore, you will want to develop a standard notebook page for each antenna you model. Here are the preliminary items that should go on the page.

Item 1

Make a neat sketch of the antenna, including all dimensions. This includes any changes in element dimensions, a common occurrence for beams using aluminum tubing.

Item 2

Tabulate the details of each element, including the total length, the length of each piece or section of wire or tubing that makes up the element, and their diameters or wire sizes. Also include the spacing between elements for multielement antennas. At this point, it may be good to place the tabulated data in a column on the left side of the notebook page, since you will want to manipulate that data before entering it into the program.

All MININEC programs require entry of the data in the form of x, y, and z coordinates for the ends of each "wire" or element section, where x is the standard left-right axis, y is the standard front-back (on the computer screen, up-down) axis, and z is the height of the element above ground. Each element section made from a different diameter wire or tube has two entries, corresponding to the two ends of the section. It makes no difference to the program whether you model the antenna using x for the element length and y for the spacing between elements or vice versa. The convention you choose depends on which system is most convenient.

Since the chief problem I have in setting up models for the computer is entry errors, I have chosen x for the antenna element lengths and y for the spacing. This puts the coordinates most likely to be numerous or to change (values for x) in the left-hand column of the entry readout, where I can survey them easily for errors. And hard-to-see errors do occur, as when I forgot a decimal point and made part of an antenna element 75 inches in diameter rather than 0.75 inches. Adopting this convention will require that we take our elevation plots at an angle of 90° rather than at 0°, as noted later on.

Let's go a step further. Although you can enter any set of x and y coordinates, so long as the element sections line up, you will probably make fewer errors if you set up your antenna symmetrically around x = 0 and y = 0. Since y is the antenna element spacing dimension, set the driven element at y = 0. A simple dipole, of course, will have no other element. A Yagi might have either or both a director and a reflector. Set the reflector behind the driven element by giving it a negative y sign. A reflector 6 feet behind a driven element shows up as y = –6 on the chart. Any directors receive positive spacing. This convention allows you to identify elements in your setup chart.

Set up your antenna elements symmetrically around x = 0. For a single-wire element, this means taking the total length and dividing it by two. The ends of the wire then have identical x entries, but one is negative while the other is positive. For example, a 66-ft dipole would have x entries of 33 and –33 for its two ends. Multisection elements of different diameters are only slightly more complex. I have a portable 10-meter dipole made from 4-foot sections of ¼-inch diameter aluminum rod with end pieces of 3⁄16-inch rod. The overall length is 16.6 feet. We can consider the center rods as one 8-foot piece, which gives us –4 and +4 for the pair of x coordinates. Each end piece is 4.3 feet long. The x coordinates for the outside ends will be –8.3 and +8.3, with the other ends of the end rods having the same x coordinates as the center rod.

There are two advantages to using this system. First, you can spot errors more easily on the element entry screen or printout. Just look for numbers that are supposed to be the same except for a sign change. (And be sure to survey all the signs for correct negative or positive values while you are at it.) Your notebook page can have compact entries in preparation for computer work.

Item 3

Enter in columns next to each element section the arithmetic you did to determine element coordinates. For example, the end element sections of the dipole might show 16.6 ft – 8 ft = 8.6/2 = 4.3 ft ±4 ft = ±8.3 ft.

Item 4

Enter the final x coordinates for each element section. If you are careful, you can use abbreviated notations. You can put sections 1 and 3 together as ±8.3 and 4.0, with section 2 as ±4.0. This means that section 1 goes from –8.3 to –4.0, section 2 from –4.0 to +4.0, and section 3 from 4.0 to 8.3. If you are not comfortable with abbreviated listings, then list each piece or section separately. Note that what we have called "pieces" or "sections" are called "wires" in the program. What the program calls "wires" may actually be wire conductors, or they may be lengths of metal tubing or rod.

Figs 1 and 2 illustrate two notebook pages, one for the simple dipole, the other for a 3-element Yagi. Notice that all the entries are in feet and decimal parts of feet. Therefore, add another item to the pencil and paper you need in order to plan your entries: a calculator. You will often encounter dimensions like 9 feet 9½ inches, which a calculator easily converts to 9.7917 feet.

Band	Antenna Type	# El.	Height	Ground	Date	Run ?
10 M	Dipole (portable)	1	20'	R	01 / 12 / 1991	x

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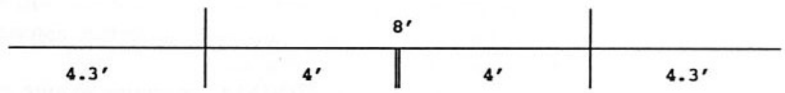

El. #	Lgth feet	Sp to DrEl	Calculations	Wire #	E1 x	y	z	E2 x	y	z	Dia.	Segs
1	16.6	-----	16.6-8=8.6/2=4.3	1	-8.3	0	20	-4	0	20	.1875	2
			4.3+/-4.0=+/-8.3	2	-4	0	20	4	0	20	.25	8
				3	4	0	20	8.3	0	20	.1875	2
Portable dipole, disassembles and stores inside 2 5-foot PVC tubes used as mast.												

Fig 1—Example notebook page for a 10-meter dipole.

Band	Antenna Type	# El.	Height	Ground	Date	Run ?
10 M	Yagi (Ant. Bk. 15th Ed p. 11-11)	3	30	R	01 / 12 / 1991	x

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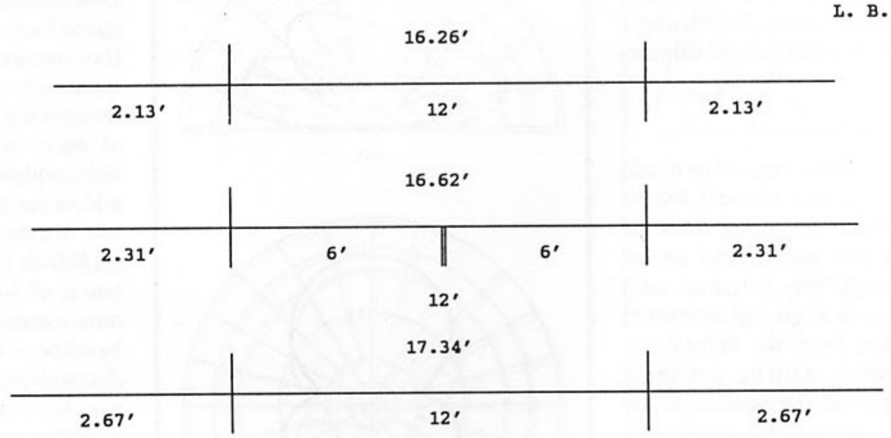

El. #	Lgth feet	Sp to DrEl	Calculations	Wire #	E1 x	y	z	E2 x	y	z	Dia.	Segs
1	16.62	---	16.62-12=4.62/2=2.31	1	-8.31	0	30	-6	0	30	.875	2
			2.31+/-6=+/-8.31	2	-6	0	30	6	0	30	1.0	8
				3	6	0	30	8.31	0	30	.875	2
2	17.34	5.23	17.34-12=5.34/2=2.67	4	-8.67	-5.23	30	-6	-5.23	30	.875	2
			2.67+/-6=+/-8.67	5	-6	-5.23	30	6	-5.23	30	1.0	8
				6	6	-5.23	30	8.67	-5.23	30	.875	2
3	16.26	3.49	16.26-12=4.26/2=2.13	7	-8.13	3.49	30	-6	3.49	30	.875	2
			2.13+/-6=+/-8.13	8	-6	3.49	30	6	3.49	30	1.0	8
				9	6	3.49	30	8.13	3.49	30	.875	2

Antenna Book design used for program test purposes.

Fig 2—Example notebook page for a 3-element Yagi.

Too, you will be using fractionally dimensioned tubing (for example, 7⁄8 inch diameter) that you need to enter as a decimal (0.875). And, of course, the calculator serves as a check on addition and subtraction.

Item 5

Multielement antennas remind us to add two more listings. For each element, list the y and z coordinates, the spacing from the driven element and the height above ground. For all single-wire antennas, such as the dipole, set y = 0. Yagis will have other elements separated from the driven element. Each element will be the same distance above ground. Remember to set the source in the center of the driven element for each of the antennas noted here. In order to ensure that the program places your source at the exact center, use an even number of wire segments for the driven element. Otherwise, your source may be offset.

Use the height that you actually anticipate the antenna will be. If you model quads, of course the vertical sections will have changing z coordinates. Once you get used to setting up a few dipoles and Yagis, the changes for vertical sections will come naturally. Start by dividing the vertical section symmetrically and add or subtract from the height of the boom or hub, which you can determine from the real or anticipated tower height.

This completes the notebook page, except for any reminders you may wish to add. I generally enter a note that tells why I modeled the antenna. That refreshes my memory weeks later when I run across the page and wonder why I ever spent computer time on that crazy design.

Building a Baseline of Antenna Information

Let's begin this part of our work with a little scenario. You model a 2-element X beam, using some guesswork and intuition for the dimensions. The program produces the patterns shown in Fig 3. Now, what have we learned? Not much, perhaps, beyond the fact that this is probably not an antenna we'd want to actually build. But you can learn much more by practicing with basic antennas over the soil and land around your own QTH.

This is the reason why, in the program setup, I suggested following the instruction manual and selecting ground conditions that most closely correspond to your own land. I chose average soil for all the examples here because everything indicates that my hilltop QTH on what once was Tennessee farmland is just that: average.

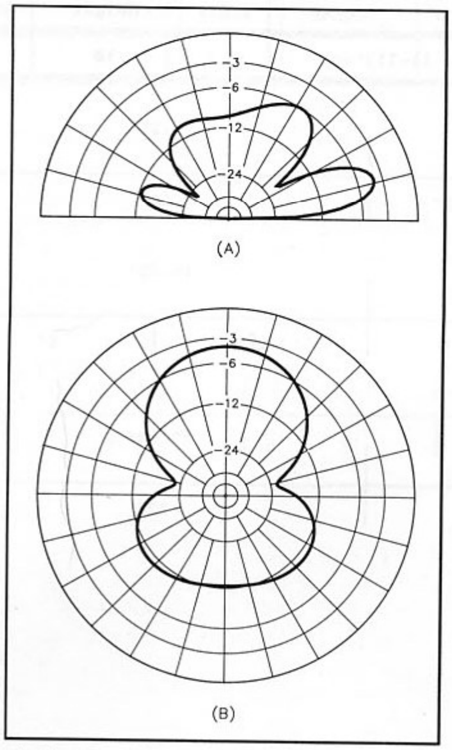

Fig 3—Elevation and azimuth patterns for a 10-meter X beam design, 30 feet high. The maximum gain is calculated to be 9.1 dBi. The azimuth pattern is taken at an elevation angle of 16°. **All patterns in this article were calculated with ELNEC 2.20 for "average" earth having a conductivity of 5 millisiemens per meter and a dielectric constant of 13. In all pattern plots, the 0-db reference (outer ring) is 13 dBi, and all azimuth patterns are taken at the elevation angle of maximum forward gain.**

Once you select and model your ground conditions, stick with them for all models, or your designs may not be directly comparable. If you discover that your soil, rock, or sand differs from your original estimates, then revise and rerun all the antennas that make up your baseline of information.

The next step is to model some basic antennas. Which antennas? You may wish to start with a dipole, a 2-element Yagi, and a 3-element Yagi, as these are all well-established antenna designs with fairly well developed characteristics. Yagi designs are available in many handbooks and ham magazines.

None of these antennas requires more than a single source, and they introduce no loads. Save those complexities until you have mastered the program basics (and then review the manual on how to use them). Moreover, all of these antennas conform to the general idea of avoiding program limitations that may produce misleading results. There are no acute angles for which special techniques are required. Likewise, none of these designs use close-spaced wires, as with folded dipoles. They, too, require special techniques to give accurate results. And none of these basic designs require three or more wires to join at one point.[3] When you are fully comfortable modeling basic designs, then you can add other techniques to your repertoire, one at a time.

Which frequency or frequencies should you use? Select the band in which you are most interested. Later you can expand your baseline to include other bands, especially if you decide to develop multiband antennas. The examples here use 28.2 MHz.

What antenna height or heights should you model? Use realistic heights relative to your situation. For convenience, I modeled the example antennas at 20 and 30 feet, since the lower height is where my temporary dipole is, and the higher elevation is about the size tower I plan to erect. Although two levels are sufficient to illustrate why the baseline data is important, expanding the baseline to 40, 50, and 60-foot heights is not unrealistic.

Heights of at least 20 feet work well for modeling at 10-meter frequencies. However, at lower frequencies (for example, 3.5 MHz), 20 feet would place these horizontal antennas well below the 0.2-λ minimum height for good modeling.[4] Table 1 provides a quick reference to the 0.2-λ height for the HF bands. Below these heights, results are likely to show incorrect

Table 1
0.2 Wavelength at the Lower End of Low-Frequency Amateur Bands

Frequency	*0.2 Wavelength*
1.8 MHz	109 feet
3.5	56
7.0	28
10.1	19.5
14.0	14
18.068	10.9
21.0	9.4*
24.89	7.9
28.0	7

*Heights below this point are not generally useful for modeling, as yard clutter may affect patterns more than program inaccuracies. Mobile, portable, and experimental antenna designs may be exceptions.

impedances and excessive gain. Always keep in mind that MININEC-based programs calculate impedances over a perfect ground, whether you select a perfect or real ground. The results are usable for real ground above the 0.2-λ limit, but may not be usable for less height. Under some circumstances, you may have to erect an antenna below the modeling limit; the antenna may work, but the program may not accurately model its operation.

The three chosen antennas at two elevations each require only six runs of the program. Using constant-diameter elements for the baseline simplifies the design work and lets the computer race through the calculations. More complex antenna construction, of course, gives you more element sections and inevitably longer calculation times.

My early experiences have sold me on the idea of beginning with an elevation pattern. (Elevation is simply the vertical radiation pattern, and azimuth is the horizontal radiation pattern.) Since the x coordinate is in line with the antenna elements, the elevation pattern requires a 90° orientation angle to catch the main lobe. The elevation pattern then tells what angle to use for the azimuth pattern to catch the perimeter of maximum radiation. You can check other azimuth patterns at other angles, but for initial information, a basic elevation and azimuth pattern combination will be instructive. For the purposes of comparison, all the antennas use the same value for the scale on which they are plotted. This permits the omission of detailed analysis information, although you may want that information on the plots you take for your own use.

Figs 4 and 5 illustrate the 10-meter dipole performance at 20- and 30-foot heights. The difference is revealing. The 30-foot-high dipole channels significant energy into high radiation angles, while the 20-foot high dipole appears to concentrate more energy at lower radiation angles. The patterns may come as a surprise if the only patterns you have viewed are free space types. Moreover, the patterns seem to supply an answer to my wondering about the excellent reports from Europe on my 20-foot high 10-meter dipole (without nearby trees). Figs 4B and 5B provide the azimuth patterns of the dipole. They show definite reductions off the ends of the antenna, but not the classic pinch-waisted "figure 8" of free-space patterns. Note that these patterns apply to 10 meters and not necessarily to other bands, because patterns will vary with (among other factors) the percentage of a wavelength the antenna is placed above ground.[5]

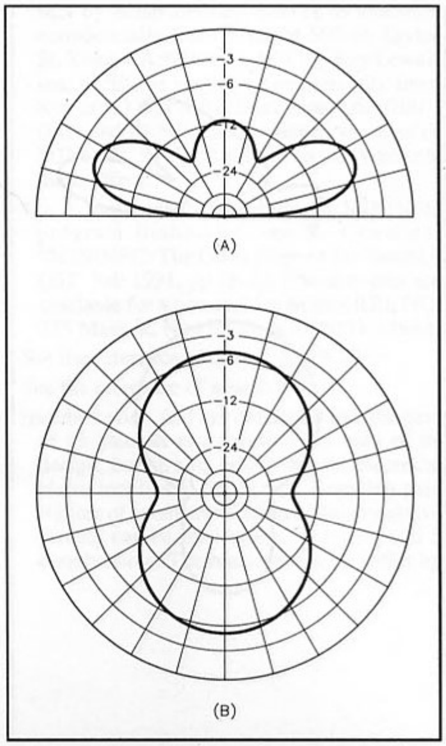

Fig 4—Patterns for a 10-meter dipole, 20 feet high. Azimuth pattern at 24° elevation; maximum gain 8.1 dBi.

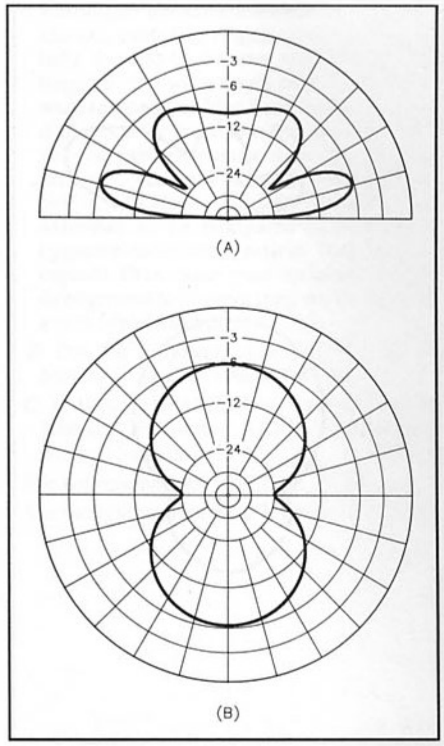

Fig 5—Patterns for a 10-meter dipole, 30 feet high. Azimuth pattern at 16° elevation; maximum gain 6.9 dBi.

It is tempting to compare the 30-foot-high dipole patterns of Fig 5 with Fig 3, the intuitive X beam at 30 feet, and notice the variation of the elevation pattern from the dipole. However, before we can evaluate whether or not the variation—which gives some gain and front-to-back ratio—is good, bad, or indifferent, let's look at the Yagis.

Figs 6 through 9 present patterns for the Yagis at 20 and 30 feet. The 2-element Yagi comes from a design by Bill Orr, W6SAI,[6] while the 3-element Yagi is adapted from *The ARRL Antenna Book*.[7] By comparing the various patterns at equivalent heights, we can see the evolution of the dipole pattern into something with gain and front-to-back ratio. Had we skipped the 2-element design, we might have missed the continuity of pattern development, which shows with special clarity at 30 feet.

The patterns reveal a number of other factors of relevance. First, the Yagis provide significant, but not overwhelming gain relative to real dipoles. (Note: ELNEC provides gain in dBi, gain over an isotropic source. Our interest is in comparing gains of real antennas over real property. To do that, we need only use the *difference* in gain figures to see if we are making significant improvements, and how much.) Only when we combine the gain with the front-to-back ratio do we find the real merits of a beam. The 3-element Yagi shows the most significant increase in front-to-back ratio among the antennas used here for baseline data. A complete analysis of our individual antenna situations would require more standard models, but we have enough here to begin generating expectations. Rational expectations are what good baseline data are designed to give us.

For example, any beam we might propose to build at selected heights should show patterns that compete with the Yagis. We can now see that the intuitive X beam at 30 feet does not do the job. Its pattern is little more than a barely deflected dipole pattern. It does not match even the 2-element Yagi for gain and front-to-back ratio. That does not mean there are not good X beam designs; rather, I came up with a bad beam antenna design and used it for this example. What told me this is a bad design was not a textbook or the program manual, but the baseline information I collected from the antenna modeling program.

The Next Step: Your Own Antennas

Your antenna situation differs from mine, which means that many of the pat-

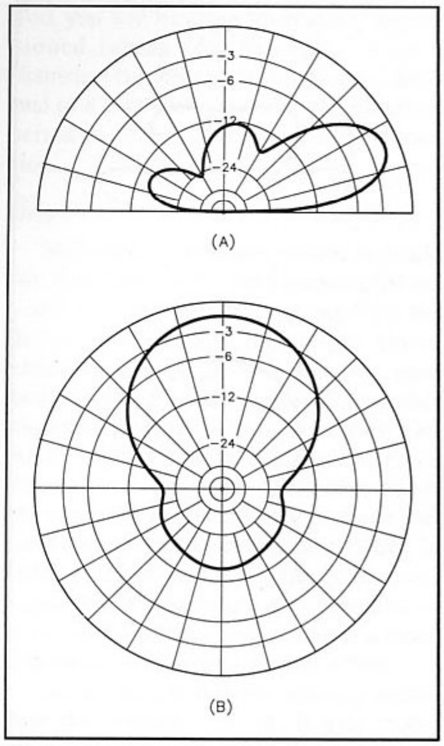

Fig 6—Patterns for a 10-meter 2-element Yagi, 20 feet high. Azimuth pattern at 23° elevation; maximum gain 11.6 dBi.

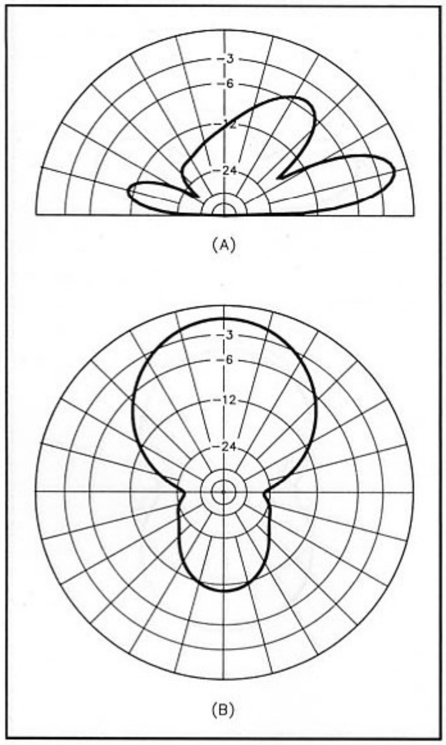

Fig 7—Patterns for a 10-meter 2-element Yagi, 30 feet high. Azimuth pattern at 16° elevation; maximum gain 11.8 dBi.

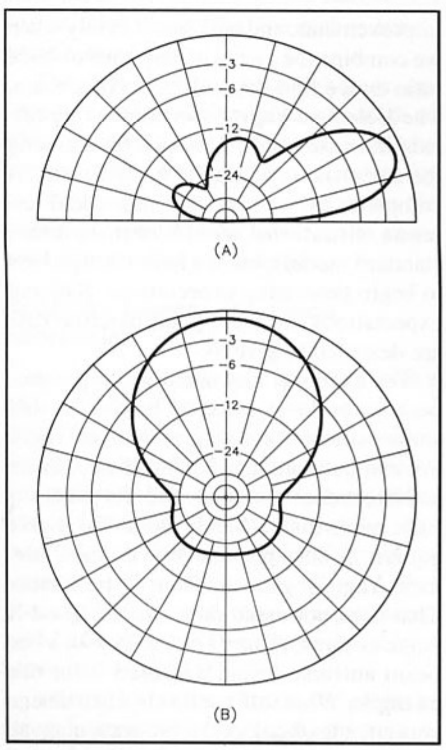

Fig 8—Patterns for a 10-meter 3-element Yagi, 20 feet high. Azimuth pattern taken at 23° elevation; maximum gain 12.3 dBi.

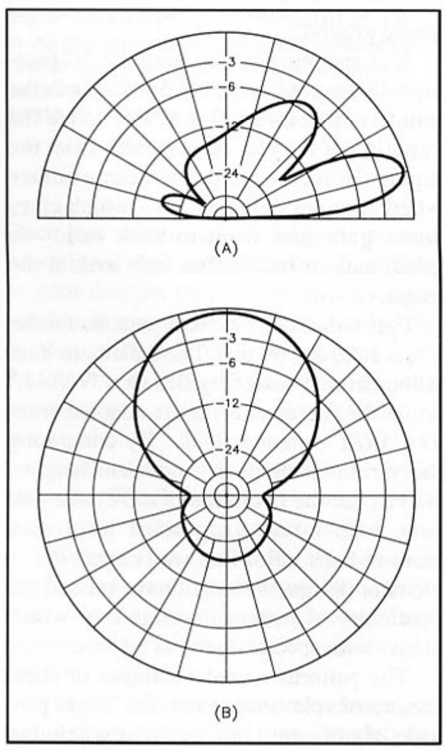

Fig 9—Patterns for a 10-meter 3-element Yagi, 30 feet high. Azimuth pattern taken at 16° elevation; maximum gain 12.6 dBi.

terns shown here may be irrelevant to you. However, they do illustrate the techniques needed by beginners to get the most out of a computer modeling program. The program can be like a textbook written for one person: you. From it, you can put theory into practice without undergoing the expense and time required to build every antenna design you encounter. You can also develop enough models to help you decide where to place your antenna dollar most wisely. Your antenna dollar includes not only the antenna itself, but the mast and tower as well. So even if you practice using the program by modeling the antennas shown here, the next step is to develop a set of models specific to your own needs. A good place to begin is with the antennas you are actually using (unless they contain traps or other complexities that may require advanced techniques to model). The assembly manuals or the calculations you performed to roll your own probably provide almost all the data you need.

Whether you choose your existing antennas or a set of proposed antennas to evaluate with the antenna modeling program, there are a few additional suggestions that may make your effort more profitable. For any antenna, be certain to evaluate it across the entire frequency band of interest. What the band of interest is depends upon your operating interests. If you use the entire band, from the lowest CW segment to the highest phone segment, then you will want antennas that are broadbanded. Their patterns may not show the most gain among your models, but they will show good gain and proper takeoff angles (and, for beams, reasonable front-to-back ratio) over the entire band. If your interests are confined to a narrow portion of any given ham band, then you should check every design at selected spots within that spectrum slice. When making comparisons between designs, use the same set of frequencies for each run.

At this point, the "Source" information becomes important. You will have to be able to match a real antenna to the feed line over the entire band of interest. A high-performance design that will not accept power is not a good antenna for you. Of course, there are many feed-point matching schemes, but choosing one goes outside the program. As you can see, an antenna modeling program is one important part—but not the only part—of an antenna design and construction project or program.

There are many other facets of using antenna modeling programs like ELNEC. The instruction manual is a good guide to them. Working with vertical antennas and ground planes or radial systems, for example, is a topic unto itself with many attached cautions.[8] Accurately modeling real ground requires reference to the program instructions and an understanding of the effects of ground upon antennas.[9] The information presented here is intended to get the beginning antenna modeler started in using the program to learn about antennas, even if he or she never builds one from scratch. Besides all these practical benefits, the programs are fun to use—especially on a rainy day when the bands are closed.

Notes

1 MININEC3 by Naval Ocean Systems Center, San Diego, is available from National Technical Information Services, Springfield, VA 22161, order no. ADA181681. This is public domain software, but a generous fee is charged for the diskette and documentation. *MN by Brian Beezley, K6STI, is available* commercially from Brian at 507-½ Taylor St, Vista, CA 92084. ELNEC by Roy Lewallen, W7EL, is available commercially from Roy at PO Box 6658, Beaverton, OR 97007. (MN and ELNEC are enhanced versions of MININEC3.) The ARRL in no way warrants these offers.

2 For a "must-read" description of MININEC program limitations, see R. Lewallen, "MININEC: The Other Edge of the Sword," *QST*, Feb 1991, pp 18-22. (Photo copies are available for a nominal fee from ARRL HQ, 225 Main St, Newington, CT 06111-1494.)

3 See the reference of note 2, pp 19-20.

4 See the reference of note 2, p 21.

5 [*Editor's note:* Always remember that the gain of an antenna is a function not only of its design, but also its height and the electrical characteristics of the earth. Comparing gain figures of antennas at different heights above ground can be misleading, as Figs 4 and 5 clearly show: The maximum gains differ by 1.2 dB, *for the same antenna!* Of course an antenna cannot have gain over itself. Similarly, front-to-back ratios may change with height at the wave angle of maximum forward response, as Figs 8 and 9 indicate. (The difference for these two heights is 2.1 dB.) As you gain experience with the modeling program, you may want to eliminate any height effects by initially modeling the antennas to be compared in free space (ground-mounted vertical systems excepted). *Then* model your optimized designs over ground to observe their performance in a real-life environment.]

6 B. Orr, "A Compact 2-Element Yagi for 10 Meters," *CQ*, Dec 1990, pp 83-84.

7 G. Hall, Ed, *The ARRL Antenna Book*, 15th or 16th eds (Newington: ARRL, 1988 or 1991), design charts on p 11-11.

8 See the reference of note 2, pp 20-22.

9 See the reference of note 7, Chapters 3 and 8.